Engineering Notes

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Optimal Switching Criteria for Two-Position Configuration Controls

Louis N. Glaros Jr.*
Martin Marietta Aerospace, Orlando, Fla.

Nomenclature

c = specific fuel consumption
D = drag
E = energy = $h + V^2/2g$
h = altitude
H = Hamiltonian in calculus of variations solution
J = performance index
L = lift
R = downrange = $\times$
t = time
T = thrust
V = velocity
W = weight

Subscripts

1 = control position 1
2 = control position 2
cr = cruise
opt = optimal

Introduction

THE use of two-position configuration controls for aerospace performance applications is quite common. Afterburners (A/B) used by jet aircraft are an example, the two positions being A/B on and A/B off. Certain high performance missiles utilize folded wings that can be deployed in flight for enhanced maneuverability or greater cruise efficiency. Also, two-position variable geometry nozzles can be used for airbreathing missiles in order to provide high accelerations at launch with a large throat and good cruise efficiency at high altitudes with a small throat.

In general, the use of these devices will increase the performance of a flight vehicle. However, the amount of improvement is strongly dependent on the manner in which they are used. Primary concerns in this area are: 1) the optimality of the trajectories flown for each configuration, and 2) the point along a given trajectory when the control is switched from one position to the other. The importance of the latter can be illustrated by considering the variable geometry nozzle (VGN). If the nozzle throat is changed from large to small too early in flight, performance will be degraded since acceleration capability is penalized and fuel consumption is consequently increased. Likewise, cruise efficiency is decreased by late switching of the VGN.

The objective of this paper is to present a number of optimal switching criteria for these two position controls. The performance measures considered are minimum time-to-climb, minimum fuel-to-climb, and maximum range with a fixed fuel weight. Performance optimization for these problems has already been well documented by a number of authors, including Schultz and Zagalsky,[1] Bryson, Desai, and Hoffman[2] and others. Their work utilizes energy management techniques to solve for optimal altitude and thrust histories as a function of energy. The solutions are quite valid and useful for a large class of flight vehicles.

The work documented in these references incorporated the energy state and quasi-steady approximations. Under these assumptions, the equations of motion for a flight vehicle moving in the vertical plane are

$$\dot{E} = (T-D)\,V/W, \quad \dot{X} = V, \quad \dot{W} = cT, \quad L = W \tag{1}$$

Optimal control solutions based on these equations involve finding the thrust and altitude histories that minimize the given performance index subject to interior and terminal constraints.

As shown in Refs. 1 and 2, the control variable thrust appears linearly in the Hamiltonian for all the performance problems cited. Consequently, for energy climb solutions, maximum thrust arcs are optimal. Altitude profiles are determined by applying the optimality condition, $\partial H/\partial h = 0$. Thus, the minimum time to climb solution is

$$h_{opt} = \max_h [(T-D)\,V/W] \tag{2}$$

while the minimum fuel to climb flight profile is defined by

$$h_{opt} = \max_h [(T-D)\,V/WcT] \tag{3}$$

and the solution for maximum range with a fixed fuel weight is

$$h_{opt} = \max_h [(T-D)/\dot{W}(-1 + (V/cT)_{cr} cT/V)] \tag{4}$$

where the cruise phase is defined by

$$(h,E)_{opt\,cr} = \max_{h,E} (V/cT) \tag{5}$$

Typical flight profiles for these problems are shown in Fig. 1. The results presented are for a ramjet propelled missile of fixed configuration (i.e., no wing deployment or VGN, etc.). As shown, this type of vehicle has a minimum time profile which occurs at high velocities and, in fact, is constrained by structural and thermal limits of the flight envelope. Minimum fuel and maximum range trajectories are flown at higher altitudes where fuel economy is greater. The dashed lines in Fig. 1 represent instantaneous altitude changes, allowed by treating altitude as a control variable.

Optimal Switching Criteria

The results of the previous work can now be extended to include the use of two-position configuration controls. The objective is to determine the energy level at which control switching should occur in order to optimize performance. Flight profiles for each control position are identical to those presented in Eqs. (2-5) while, at the time of switching, rapid climb or dive phases may result in order to piece the trajectories together.

The optimal switching energy can be determined by first deriving a relationship for the performance index as a function of energy, and second, using the theory of maxima and minima to establish the location of the stationary point of this function. In general, the performance index can be written as

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*Senior Engineer. Member AIAA.

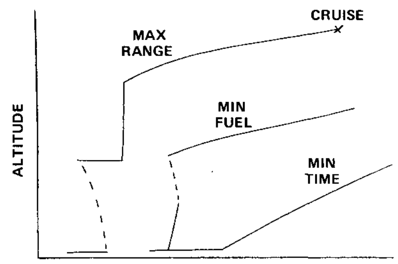

Fig. 1 Energy management solutions to certain performance problems.

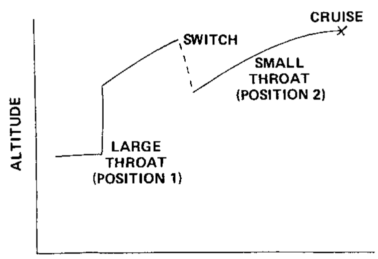

Fig. 2 Maximum range energy management trajectory for a ramjet missile with a VGN.

a sum of performance due to both control positions, $J(E_{sw}) = J_1 + J_2$. Differentiating with respect to energy and setting the result equal to zero determines the switching point for extreme values of the performance index. That is, $\mathrm{d}J(E_{sw})/\mathrm{d}E = \mathrm{d}J_1/\mathrm{d}E + \mathrm{d}J_2/\mathrm{d}E = 0$ implies E_{sw}. By substituting Eq. (1), this stationary condition becomes a function of vehicle performance parameters. In fact, switching depends only on point performance capabilities of the two configuration control positions, and not on boundary conditions of the problem.

If the derivative of the performance index, $\mathrm{d}J(E_{sw})/\mathrm{d}E$, does not reach zero within the flight envelope, then the extreme value of the function occurs at either the initial or final energy level. The exact location will depend on whether the problem involves maximization or minimization and on the sign of the derivative, $\mathrm{d}J/\mathrm{d}E$.

Applying this procedure to a specific performance problem will result in an optimal switching criterion for that problem. For minimum time to climb, optimal switching will occur when the energy rates for both control positions are equal

$$(T-D)V/W|_1 = (T-D)V/W|_2 \tag{6}$$

For minimum fuel to climb, the switching criteria is

$$(T-D)V/WcT|_1 = (T-D)V/WcT|_2 \tag{7}$$

which indicates that, when the energy gain per pound of fuel used for control positions 1 and 2 are equal, switching should occur.

Maximum range trajectories are assumed to be flown with the control in position 2 for the cruise phase. Thus, the optimal switching criterion is

$$(W/(T-D) - (V/cT)_{cr_2}[WcT/V(T-D)]|_1$$

$$(W/(T-D) - (V/cT)_{cr_2}[WcT/V(T-D)]|_2 \tag{8}$$

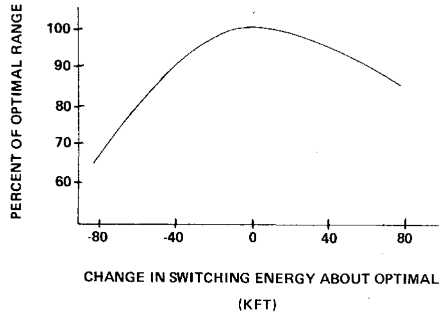

Fig. 3 Range attainable for variations in switching energy.

The switching criteria of Eqs. (6-8) also follow from an application of the calculus of variations approach of Refs. 1 and 2. In this case, use is made of the first integral condition, H = constant, and it then follows that Eq. (8) is valid for trajectories without any cruise phase, indicated by replacing $(V/cT)_{cr_2}$ by (V/cT) evaluated at the final time.

Example Problem

The preceding solution was applied to the performance optimization of a ramjet propelled missile utilizing a variable geometry nozzle. In this case, control position 1 corresponded to a large throat area and position 2 to a small throat; the ratio between the two being 1.47. The problem considered was maximum range, and flight profiles for each configuration were found using Eqs. (4 and 5). The optimal switching energy was determined by application of Eq. (8) and was found to be approximately 110 kft below cruise conditions. The optimal flight profile for the problem is shown in Fig. 2. Included is the trajectory for each configuration as well as the transition phase between controls. Since the large initial throat area requires high altitudes, at the switch point a rapid loss in altitude is necessary in order to place the vehicle on the flight profile corresponding to the small throat.

The optimality of the final result can be seen in Fig. 3, where percentage range (compared to the range at the optimal switch point) is plotted against change in switching energy about the optimal. This figure shows that the criterion of Eq. (8) did provide the maximum range switch point. Here it can be seen that switching too early causes a rapid loss in attainable range, about 15% at $\Delta E_{sw} = -50$ kft; whereas switching too late is not quite as degrading, $\Delta E_{sw} = 50$ kft penalizing the range by only 8%.

Conclusions

The results of previous flight vehicle performance optimization work have been extended to include the use of two-position configurations controls. Optimal switching criteria for three performance problems, minimum time or minimum fuel to climb and maximum range with a fixed fuel weight, have been established. These switching criteria depend only on the point performance capabilities of the vehicle and are completely independent of the boundary conditions. An example maximum range problem for a ramjet missile with a VGN showed that the derived criteria indeed yields optimal results.

References

[1]Schultz, R.L. and Zagalsky, N.R., "Aircraft Performance Optimization," *Journal of Aircraft*, Vol. 9, Feb. 1972, pp. 108-114.

[2]Bryson, A.E., Desai, M.N., and Hoffman, W.C., "Energy State Approximation in Performance Optimization of Supersonic Aircraft," *Journal of Aircraft*, Vol. 6, Nov.-Dec. 1969, pp. 481-488.